ON CONDENSED MILK.

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The object in analyzing condensed milk is to find whether this product is derived from whole or skimmed milk. This knowledge is obtained by reconstituting the original milk, and in so doing one at least of the constituents of milk must be known, otherwise the problem would be indeterminate.

Let a, b, c, d, e, be respectively the water, fat, albuminoids, milk sugar and ash contained in a milk, these constituents together weighing 100 grms.

If now we add to this milk x grms. of cane sugar and evaporate the original water a, a quantity represented by ay (y fractional) it is evident that b, c, d, e have not changed in absolute weights, and that the condensed milk obtained from the 100 grms. of the original milk will weigh now

$$100+x-ay.$$

Since the analysis gives for the different constituents, per cent., new values a', b', c', d', e' in the condensed milk, it is evident that

b of fat, on $100+x-ay$ total weight of condensed milk, represents on 1 of condensed milk

$$\frac{b}{100+x-ay}$$

and on 100

$$\frac{100\,b}{100+x-ay}=b'$$

b' being the percentage of fat in condensed milk, and in the same manner we will have for any other constituent, θ, in the original milk

$$\frac{100\ \theta}{100+x-ay}=\theta'$$

Making $100+x-ay=D$, and since in general $\theta'=\frac{100\ \theta}{100+x-ay}$ we have

$$\theta'=\frac{100\ \theta}{D}$$

and

$$\theta=\frac{D\ \theta'}{100}$$

Consequently

$$b=\frac{D\ b'}{100}$$

$$c=\frac{D\ c'}{100}$$

$$d=\frac{D\ d'}{100}$$

$$e=\frac{D\ e'}{100}$$

This gives, immediately, $100\ \theta=D\ \theta'$ and $D=\frac{100\ \theta}{\theta'}$ and the denominator D can be found at once, as soon as we assume θ to be known in the original milk, by assuming, *a priori*, any one of the constituents, *a, b, c, d, e*.

If we assume *d*, then

$$D=\frac{100\ d}{d'}$$

and, knowing D, we can find readily *b, c, e* and *a* by difference from 100.

All the constituents of the original milk then are found without any difficulty by the simplest equations.

APPLICATION OF THE PRECEDING FORMULÆ.

Found in condensed milk:

a′	Water	25.47
b′	Fat	10.05
c′	Albuminoids	9.36
d′	Milk sugar	10.19
e′	Ash	1.92
x	Cane sugar	40.01
		100

Assuming that milk sugar = 4 p. c. in normal milk, we have

$$d=\frac{D\,d'}{100},\quad D=\frac{100\,d}{d'}=\frac{100\times 4}{10.19}=39.25$$

and

$$b=\frac{39.25\times 10.05}{100}=3.94 \text{ per cent. Fat in original milk.}$$

$$c=\frac{39.25\times 9.36}{100}=3.67 \text{ " Albuminoids " "}$$

$$d=\frac{39.25\times 10.19}{100}=4.00 \text{ " Milk sugar " "}$$

$$e=\frac{39.25\times 1.92}{100}=0.75 \text{ " Ash " "}$$

12.36 " Solids " "
87.64 " Water " "

Assuming respectively for *b* or *c* or *d* or *e* values increasing by 0.1, what will be the composition of the original milk calculated on these different assumptions and using the preceding formulæ.

Composition of an original milk:

a	Water	86.87
b	Fat	3.50
c	Albuminoids	4.92
d	Milk sugar	4.00
e	Ash	0.70
		99.99

to which we add 12.00 of cane sugar.

Water	86.87
Fat	3.50
Albuminoids	4.92
Milk sugar	4.00
Ash	0.70
Cane sugar	12.00
	111.99

or in per cent.

Water	77.57
Fat	3.12
Albuminoids	4.39
Milk sugar	3.57
Ash	0.63
Cane sugar	10.72
	100.00

Further we concentrate to 90 per cent. of the original quantity of water.

Water	7.76
Fat	3.12
Albuminoids	4.39
Milk sugar	3.57
Ash	0.63
Cane sugar	10.72
	30.19

The composition per cent. of the condensed milk will then be :

a' Water	25.70
b' Fat	10.35
c' Albuminoids	14.54
d' Milk sugar	11.83
e' Ash	2.08
x Cane sugar	35.50
	100.00

Then we know to a certainty that this condensed milk comes from an original milk of known composition and in assuming for *b*, *c*, *d*, *e* the values given we shall, from the values *a'*, *b'*, *c'*, *etc.*, of the condensed milk, find the composition of this original milk.

We have d=4.00

$$D=\frac{100\,d}{d'}=\frac{100\times 4}{11.85}=33.81$$

and

$$b=\frac{33.81\times 10.35}{100}=3.50 \text{ per cent. Fat in original milk.}$$

$$c=\frac{33.81\times 14.54}{100}=4.92 \text{ `` Albuminoids `` ``}$$

$$d=\frac{33.81\times 11.83}{100}=4.00 \text{ `` Milk sugar `` ``}$$

$$e=\frac{33.81\times 2.08}{100}=0.70 \text{ `` Ash `` ``}$$

13.12	"	Solids	"	"
86.88	"	Water	"	"

As a verification the water *a* could be obtained directly.

Since we have

$$100+x-ay=D$$

ay=100+x—D = 112—33.81= 78.19, the weight of water evaporated, then there remains a—ay water in the original milk on a total weight D of condensed milk, corresponding in 100 to

$$\frac{100\,(a-ay)}{D}=a'$$

whence

$$a=ay+\frac{Da'}{100}$$

or

$$a=78.19+\frac{33.81\times 25.70}{100}=86.88$$

The following table shows the increase of 0.1 for *b*, *c*, *etc.*, that is, instead of *b*=3.50, *b*=3.60. Instead of *c*=4.92, *c*=5.02, *etc.*

ORIGINAL MILK.	PROCEEDING BY INCREASE OF 0.1 PER CENT. AND ASSUMING			
	d	b	c	e
a=86.87	86.55	86.50	86.59	84.99
b= *3.50*	3.586	*3.60*	3.77	4.00
c= *4.92*	5.04	5.06	*5.02*	5.62
d= *4.00*	*4.10*	4.11	4.08	4.58
e= *0.70*	0.716	0.72	0.71	*0.80*
x=12.00	12.30	12.34	12.24	13.72
D=33.81	34.65	34.78	35.27	38.65

We see that assuming *e*, the ash is out of the question ; its percentage being always small, it is the constituent for which the smallest difference has the greater results.

c, *b* and *d* appear to be the constituents of which the variations have the least influence, in fact any of these three assumptions of increase of 0.1 per cent. in *c*, *b* and *d* does not practically affect the composition of the original milk these constituents entering for nearly 4 per cent. each in the milk. Then we are limited in our choice to the one of these constituents of which the variations being the least, the average is the nearest to a constant. If milk sugar is such a constituent, as it appears to be, it is the one to be assumed.

To verify the correctness of the preceding formulæ, let us take again the original milk.

a	Water	86.87
b	Fat	3.50
c	Albuminoids	4.92
d	Milk sugar	4.00
e	Ash	0.70
		99.99

from which, after the addition of cane sugar and the evaporation of water, the following condensed milk has been obtained.

a′	Water	25.70
b′	Fat	10.35
c′	Albuminoids	14.54
d′	Milk sugar	11.83
e′	Ash	2.08
x	Cane sugar	35.50
		100.00

After the addition of *x* of cane sugar to the normal milk and the evaporation of *ay* of water, the mixture weighs

$$100+x-ay=100+x-86.87\ y$$

and there is left a—ay=a(1—y) water in the condensed milk, and it contains

Water	a(1—y)=86.87 (1—y)
Fat	b′
Albuminoids	c′
Milk sugar	d′
Ash	e′
Cane sugar	x

In the condensed milk weighing 100+x—86.87 y we have

$$a(1-y)\text{ water}$$

and in 100

$$\frac{100\ a\ (1-y)}{100+x-86.87\ y}$$

and we have then

a′ Water $\dfrac{86.87-86.87\ y}{100+x-86.87y}=25.70$

b′ Fat $\dfrac{100\ b}{100+x-86.87\ y}=10.35$

c′ Albuminoids $\dfrac{100\ c}{100+x-86.87\ y}=14.54$

d′ Milk sugar $\dfrac{100\ d}{100+x-86.87\ y}=11.83$

e′ Ash $\dfrac{100\ e}{100+x-86.87\ y}=2.08$

x Cane sugar $\dfrac{100\ x}{100+x-86.87\ y}=35.50$

To find the values of x and y we can operate upon any of these equations, since we know the composition of the original milk.

Taking $d=4$ and $b=3.50$ we have

$$350=1035+10.35\,x-899.10\,y$$
$$400=1183+11.83\,x-1027.67\,y$$

whence

$$y=0.811 \text{ and } x=4.26$$

and the common denominator will be

$$104.26-70,45=33.81$$

as we had found before by a shorter method.

As we have remarked, it is indispensable to know at least one of the constituents of the original milk to be able to calculate its composition from the values obtained from the condensed milk.

If in the milk under consideration we had assumed only that it contains 86.87 per cent. of water it is evident that from the equations

$$\frac{8687-8687\,y}{100+x-86.87\,y}=25.70$$

and

$$\frac{100\,x}{100+x-86.87\,y}=35.50$$

we could obtain the composition of the original milk.

Resolved, these equations give

$$x=12 \text{ and } y=0.9$$

and as before 31.81 for factor.

If we do not assume any known quantity in the original milk the equation for water is

$$\frac{100\,a\,(1-y)}{100+x-ay}=25.70.$$

and the problem is indeterminate.